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BOOK REVIEWS

Book Review Editor: Walter Van Assche

Books

Ward Cheney and Will Light, *A Course in Approximation Theory*, Brooks/Cole, Pacific Grove, 2000, xiv + 359 pp.

Introductory books in approximation theory are relatively rare. During the past 20 years we have seen the appearance of *An Introduction to the Approximation of Functions*, by T. J. Rivlin, *Approximation Theory and Methods*, by M. J. D. Powell, both from 1981, and *Constructive Approximation*, by R. A. DeVore and G. G. Lorentz from 1993, as books that cover general basic aspects of approximation theory. Most other recent books deal with more advanced or specialized topics. Hence, as a new introductory book the present book, written by two distinguished experts in the field, fills a definite gap in the literature.

The book consists of 36 chapters. Most of the chapters are short, or even very short (5 pages). The emphasis of the book is on multivariate interpolation and approximation, although univariate theory is necessarily touched upon as well. The 10 starting chapters focus on polynomial interpolation. These chapters are easy to follow, since the concepts are carefully explained and motivated, and the mathematical techniques are basic linear algebra and analysis. The following chapters deal with interpolation by translates of a single function, by positive definite functions, and by radial basis functions. Here the mathematical tools are more demanding. Use is made of Fourier analysis, measure theory, and functional analysis. The book goes on to discuss approximation by positive definite functions and ridge functions, neural networks, optimal recovery of functions, various types of spline functions, and (univariate) wavelets.

The book is easy to read. The treatment of each topic is necessarily limited, but the authors have been successful in providing an introduction to the main ideas and techniques. The book is recommended as a text for a graduate course in approximation theory. Every chapter is accompanied by a large collection of problems, as well as its own list of references, thus making the book a source of information for (beginning) researchers who want to find their way in a certain area.

The future will tell whether this book will become as influential as Cheney's classical monograph *Introduction to Approximation Theory*, that already dates back to 1966.

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C. F. Dunkl and Y. Xu, *Orthogonal Polynomials of Several Variables*, Encyclopedia of Mathematics and Its Applications **81**, Cambridge University Press, Cambridge, UK, 2001, xv + 390 pp.

The theory of orthogonal polynomials of one variable is a classical and well documented subject in mathematical analysis. In this theory the Jacobi, Laguerre, and Hermite polynomials, known as the classical orthogonal polynomials, play a special role due to their wide variety of applications. Much less literature is available on the theory of orthogonal polynomials of several variables. This subject has not progressed as quickly as the one-variable theory, probably due to the complication that a basis of orthogonal polynomials is no longer uniquely defined. Despite this problem, there have been exciting new developments in this field of research, some of which are treated in detail in the book under review.

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The book starts with a short survey of the theory of orthogonal polynomials of one variable, with special emphasis on the classical orthogonal polynomials. The second chapter contains some basic examples of orthogonal polynomials of several variables, such as spherical harmonics and the classical orthogonal polynomials on the ball and simplex. These examples are very convenient to have in mind when reading the third chapter, which deals with the general theory of orthogonal polynomials of several variables. In this chapter a "basis independent" theory of orthogonal polynomials of several variables is developed, which in particular leads to analogs of the three-term recurrence relation and Favard's theorem.

The remaining chapters of the book mainly deal with explicit examples of orthogonal polynomials of several variables with respect to reflection-invariant measures. In all these examples an essential role is played by Dunkl's rational differential-reflection operators (which generalize the partial differentiation operators $\partial/\partial x_i$).

Chapter 4 contains a short introduction to finite reflection groups and an introduction to Dunkl's differential-reflection operators. Furthermore, the intertwining operator (which maps $\partial/\partial x_i$ to the associated Dunkl operator) and the generalized exponential function are introduced. The sum of the squares of the Dunkl operators generalizes the Laplace operator and thus naturally leads to the theory of generalized spherical harmonics (called h-harmonics in the book). This topic is treated in Chapter 5, as well as the related Poisson kernels. At the end of the chapter analogs of the Fourier transform are discussed.

Orthogonal polynomials of several variables are considered to be classical if they satisfy a second-order differential-reflection equation. In Chapter 6 some examples of classical orthogonal polynomials of several variables are discussed, such as the generalized Hermite and Laguerre polynomials, and generalized classical orthogonal polynomials on the ball and simplex. Chapter 7 deals with the summability of orthogonal expansions for the h-harmonics and for the generalized classical orthogonal polynomials.

In Chapter 8 the (non)symmetric Jack polynomials are studied with respect to several different scalar products. The Dunkl operators associated with the symmetric group play an essential role in constructing a large family of self-adjoint, commuting operators for which the (non)symmetric Jack polynomials are joint eigenfunctions. The extension of the theory to the setting of the octahedral groups is discussed in Chapter 9.

The authors have restricted their attention to the orthogonal polynomials of several variables associated with rational Dunkl operators. In particular, the generalized Jacobi polynomials of Heckman and Opdam, which are associated with trigonometric-type Dunkl operators, are not discussed. Furthermore, q-deformations of the orthogonal polynomials (such as the Macdonald polynomials) are not treated in the book. The connection of orthogonal polynomials of several variables with completely integrable quantum many-body systems is the only application which is discussed in detail.

This book is a valuable addition to the literature, especially since it is the first detailed and modern treatment on the theory of orthogonal polynomials of several variables. It is reasonably self-contained and carefully written. There are several typos in the book, but most of them are harmless. The book is also well suited for non-specialists. Undoubtedly it will be very useful to anyone interested in orthogonal polynomials of several variables.

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Gary D. Knott, *Interpolating Cubic Splines*, Progress in Computer Science and Applied Logic **18**, Birkhäuser, Boston, 2000, xii + 244 pp.

A book with a mathematical topic is generally a text that a reader (or some reader) should be able to learn something from. If it is a good book, then the learning should also be connected with some joy. How about the book under review?